647510 **BSc5MatC503x** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov - 2021 (NEW COURSE)
Boolean Algebra & Complex Analysis -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

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1 (A) Answer any two: **(10)**

(1) Define: Equivalence class of an element. If $A \neq \emptyset$ and R is an equivalence relation defined in A and $a, b \in A$ then prove that

(i) $[a] = [b] \Leftrightarrow aRb$

(ii) either $[a] = [b]$ or $[a] \cap [b] = \emptyset$.

(2) Define: Lattice as a partial ordered set. If $(L, \leq)$ is a lattice and $a, b \in L$ then prove that

$$a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$$

(3) Prove that every chain is a distributive lattice.

(B) Answer any one: **(04)**

(1) If $L = \{1,2,3,30,42,210\}$ then show that (L, D) is a partial ordered set but not lattice. Draw its Hasse diagram and also determine its maximal and minimal elements.

(2) Define: (i) Bounded lattice (ii) Complemented lattice.
Find out the complement of each element of the lattice $(S_{30}, *\oplus)$.

2 (A) Answer any two: **(10)**

(1) Define: Atoms in Boolean algebra. Prove that a non-zero element of Boolean algebra $(B, *, \oplus, ', 0, 1)$ is an atom iff for each $x \in B$ either $a * x = 0$ or $a * x = a$.

(2) State and prove Stone's representation theorem for Boolean algebra.

(3) If $(B, *, \oplus, ', 0, 1)$ is Boolean algebra and $a, b \in B$ then prove that:
$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b = 1$.

(B) Answer any one: **(04)**

(1) Express $\alpha(x_1, x_2, x_3) = x_1 \oplus x_2' \oplus x_3$ as the sum of product canonical form.

(2) Define: Cube array representation of Boolean function.
Find the cube array representation of $f(x, y, z) = xy + xz'$.

3 (A) Answer any two: **(10)**

(1) Define: Harmonic Function. Show that $u = r^2 cos2\theta$ is harmonic
Also find an analytic function whose real Part is $u = r^2 cos2\theta$.

(2) If $f(z) = u(x, y) + iv(x, y)$ is a complex function and $z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$ are fixed complex constants and $z = x + iy$ is complex variable then prove that
$\lim_{z \to z_0} f(z) = w_0$ iff $\lim_{(x,y) \to (x_0,y_0)} u(x, y) = u_0$ and $\lim_{(x,y) \to (x_0,y_0)} v(x, y) = v_0$.

(3) If $f(z) = u + iv$ is an analytic function then in usual notations prove that

$$\nabla^2 |f(z)|^2 = 4\, |f'(z)|^2.$$

(B) Answer any one: **(04)**

(1) Define: Analytic Function. Prove that an analytic function of a constant modulus is also a constant in its domain.

(2) Find an Analytic function $f(z) = u + iv$ such that
$u - v = (x - y)(x^2 + 4xy + y^2)$.

4 (A) Answer any two: **(10)**

(1) State Green's theorem. State and Prove Cauchy's Fundamental theorem.

(2) Define: Smooth arc and state and prove ML-inequality for complex function $f(z)$.

(3) Define: Contour. In Usual notation prove that
$\int_c f(z)dz = ir_0 \int_0^{2\pi} f(z_0 + re^{-i\theta}) e^{i\theta} d\theta$, where
$c: z - z_0 = r_0 e^{i\theta}$ *and hence deduce that* $\int_c \frac{dz}{z - z_0} = 2\pi i$.

(B) Answer any one: **(04)**

(1) Find the value of Integral for the given counter C

(a) $\int_c \frac{e^{-z}}{z-\frac{\pi i}{2}}\, dz$, C : $|z|$=2 (b) $\int_c \frac{5z-2}{z(z-3)}\, dz$, C : $\left|z+\frac{1}{2}\right| = \frac{3}{2}$

(2) Evaluate : $\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz.$

5 (A) Answer any two: **(10)**

(1) State Cauchy-inequality and State and Prove Liouville's theorem.

(2) State and Prove Cauchy's Integral formula for derivative and deduce that $f^n(z_0) = \frac{n!}{2\pi i}\int_c \frac{f(z)}{(z-z_0)^{n+1}} dz.$

(3) State and prove fundamental theorem of algebra.

(B) Answer any one: **(04)**

(1) Find $\int_c \frac{e^{2z}}{(z-1)^4}\, dz$, C: $|z|$=3

(2)Find $\int_c \frac{sin^6 z}{(z-\frac{\pi}{6})^3}\, dz$, C: $|z|$=1

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